Roll Number		

SET NO. 2

INDIAN SCHOOL MUSCAT
FIRST TERM EXAMINATION
MATHEMATICS

CLASS: XII | Sub Code: 041 | Time Allotted: 3 Hrs

02.05.2018 | Max Marks: 100

General Instructions: This question paper consists of 29 questions divided into 4 sections.
Section A contains 4 questions of **one** mark each.
Section B has 8 questions of **two** marks each.
Section C contains 11questions of **four** marks each.
Section D has 6 questions of **six** marks each.

SECTION-A (4 x 1 = 4 marks)

1. Evaluate : $tan\left(2\ \tan^{-1}\frac{1}{5}\right)$
2. If $f(x) = [x]$ and $g(x) = |x|$, find $fog\left(\frac{-5}{2}\right)$
3. Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$
4. Differentiate $\sin(\cos(x^2))$ with respect to x.

SECTION- B (8 x 2 = 16 marks)

5. Let $*$ be a binary operation on the set of all non-zero real numbers given by $a*b = a+b-ab$ for all $a,b \in R-\{0\}$. Find the value of x, given that $2*(x*5)=13$.
6. Find $\frac{dy}{dx}$, if $y = cosx.cos2x.cos3x$
7. Verify Rolle's theorem for the function $f(x)=x^2+2x-8,\ x \in [-4, 2]$
8. Let $*$ be a binary operation on the set N, of natural numbers, defined by $a*b=a+b+10$, for $a,b \in N$. Find the identity element for $*$, if exists.
9. Find the point at which the line $y=x+1$ is a tangent to the curve $y^2=4x$.
10. Find $\frac{dy}{dx}$, if $x^2+xy+y^2=100$.
11. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right),\ \frac{-\pi}{2} < x < \frac{3\pi}{2}$ in simplest form.

12. Show that the function $f(x)=3x^2+5$ is differentiable at $x=2$.

SECTION- C (11 x 4 = 44 marks)

13. Differentiate w.r.t x: $(\log x)^x + x^{\log x}$

14. If the function $f:R\to R$ be given by $f(x)=x^2+2$ and $g:R\to R$ be given by $g(x)=\frac{x}{x-1}$, $x\neq 1$, find fog and gof and hence find $fog(2)$ and $gof(-3)$.

15. Show that $f:R-\{-1\}\to R-\{1\}$, given by $f(x)=\frac{x-3}{x+1}$ is a bijective function.

16. If $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & if\ x\neq\frac{\pi}{2}\\ 5, & if\ x=\frac{\pi}{2}\end{cases}$ is continuous function at $=\frac{\pi}{2}$, find the value of k.

OR

Find the value of k for which the function $f(x)=\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & if\ -1\le x<0\\ \frac{2x+1}{x-1}, & if\ 0\le x\le 1\end{cases}$ is continuous at $x=0$.

17. If $y=3\cos(\log x)+4\sin(\log x)$, show that $x^2y_2+xy_1+y=0$.

OR

If $y=\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$, show that $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-2=0$.

18. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

19. Find the equation of all lines having slope 2 and which are tangents to the curve $y+\frac{2}{x-3}=0$.

OR

Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=2$ at (1, 1).

20. Solve for x: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

21. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{d^2y}{dx^2}$.

22. If $\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}\left(\frac{y}{b}\right)=\alpha$, prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$

23. The length x of a rectangle is decreasing at the rate of 3 cm/min and the width y is increasing at the rate of 2 cm/min. When x = 10 cm and y = 6 cm, find the rates of change of (i) the perimeter and (ii) the area of the rectangle.

SECTION-D (6 x 6 = 36 marks)

24. Find $\frac{dy}{dx}$, if $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$ (OR)

If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

25. Find the intervals in which the function $f(x)=3x^4-4x^3-12x^2+5$ is
(i) strictly increasing (ii) strictly decreasing

26. An operation $\otimes$ is defined on the set $R-\{-1\}$ by $x\otimes y=x+y+xy,\ \ x,y\in R$. (i) Prove that $\otimes$ is binary. (ii) Is $\otimes$ commutative? (iii) Is $\otimes$ associative ? (iv) Find its identity element and (v) find the inverse of each element x. (vi) Also evaluate $5\otimes 3$.

OR

Prove that the relation R in the set $A=\{1,2,3,4,5\}$ given by $R=\{(a,b):|a-b|$ is an even number$\}$ is an equivalence relation.

27. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, prove that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$

28. Consider a function $f: A\rightarrow[-6,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible. Find $f^{-1}(x)$.

29. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

A wire of length 28 m is to be cut into two pieces. One of the piece is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

End of the Question Paper